Electric Scalar Potential (Voltage)

Force to move charge q against electric field $\vec{E}$

$$\vec{F}_{ext} = -\vec{F}_e = -q\vec{E}$$

Work (energy) to move distance $d\vec{l}$

$$dW = \vec{F}_{ext} \cdot d\vec{l} = -q\vec{E} \cdot d\vec{l}$$

$$dV = \frac{dW}{q} = -\vec{E} \cdot d\vec{l} \quad \text{in Volts or } \frac{\text{Joules}}{\text{Coulomb}}$$

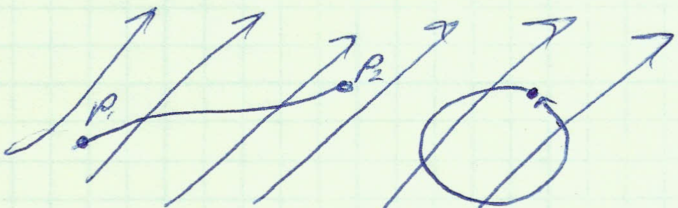
Electric field is in Volts/meter

Potential difference $V_{21} = V_2 - V_1 = -\int_{P_1}^{P_2} \vec{E} \cdot d\vec{l}$

voltage difference is independent of path chosen

$$\oint_C \vec{E} \cdot d\vec{l} = 0$$

$\nabla \times E = 0$
(electrostatics)



We often take "ground" to be a point at infinity

$$V = -\int_{\infty}^{P} \vec{E} \cdot d\vec{l}$$

Potential due to point charges

$$\vec{E} = \hat{R}\frac{q}{4\pi\epsilon R^2} \rightarrow V = -\int_{\infty}^{R} \left(\hat{R}\frac{q}{4\pi\epsilon R^2}\right) \cdot \hat{R}\,dR = \frac{q}{4\pi\epsilon R}$$

$$\boxed{V = \frac{1}{4\pi\epsilon}\sum_{i=1}^{N} \frac{q_i}{|\vec{R} - \vec{R}_i|}}$$

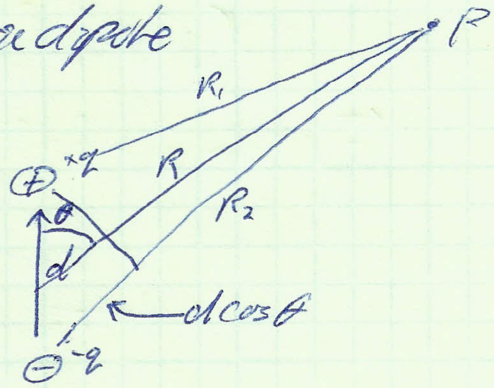
Potential due to charge distribution:

$$V = \frac{1}{4\pi\epsilon}\int_{V'} \frac{\rho_v}{R'}dV' \text{ or } V = \frac{1}{4\pi\epsilon}\int_{S'} \frac{\rho_s}{R'}ds' \text{ or } V = \frac{1}{4\pi\epsilon}\int_{l'} \frac{\rho_l}{R'}dl'$$

Electric field as function of potential:

$$\vec{E} = -\nabla V$$

Example: Electric field due to electric dipole



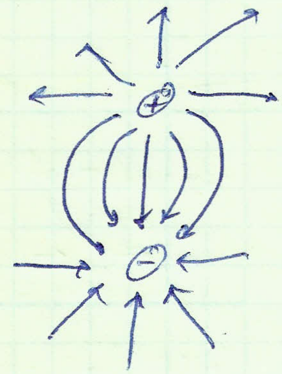
Assume $R \gg d$

$$V = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{R_1} + \frac{-q}{R_2}\right) = \frac{q}{4\pi\epsilon_0}\left(\frac{R_2 - R_1}{R_1 R_2}\right)$$

$$R_2 - R_1 \approx d\cos\theta$$

$$R_1 R_2 \approx R^2$$

$$V = \frac{q d \cos\theta}{4\pi\epsilon_0 R^2}$$



$$q d\cos\theta = q\vec{d}\cdot\hat{R} = \vec{p}\cdot\hat{R}$$

($\vec{p}$ is dipole moment, $\vec{p} = q\vec{d}$)

$$\boxed{V = \frac{\vec{p}\cdot\hat{R}}{4\pi\epsilon_0 R^2}}$$

$$\vec{E} = -\nabla V = -\left(\hat{R}\frac{\partial V}{\partial R} + \hat{\theta}\frac{1}{R}\frac{\partial V}{\partial \theta} + \hat{\phi}\frac{1}{R\sin\theta}\frac{\partial V}{\partial \phi}\right)$$

$$\boxed{\vec{E} = \frac{qd}{4\pi\epsilon_0 R^3}\left(\hat{R}\,2\cos\theta + \hat{\theta}\sin\theta\right)}$$

Poisson's Equation

from $\nabla\cdot\vec{E} = \frac{\rho_v}{\epsilon}$ and $\vec{E} = -\nabla V \rightarrow \boxed{\nabla^2 V = \frac{-\rho_v}{\epsilon}}$

Conductors

$\vec{J} = \sigma \vec{E}$ Ohm's law

perfect dielectric: $\sigma = 0$, perfect conductor: $\sigma = \infty$
$\vec{J} = 0$ $\vec{E} = 0$

- Drift velocity

$\vec{u}_e = -\mu_e \vec{E}$ m/s

μ_e is electron mobility $m^2/V\cdot s$

$\vec{u}_h = \mu_h \vec{E}$

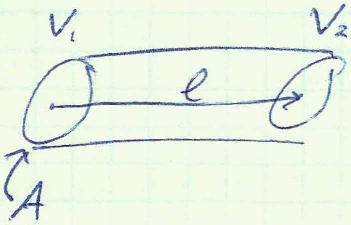
μ_h is hole mobility (for semiconductors)

$$\vec{J} = \vec{J}_e + \vec{J}_h = \rho_{ve}\vec{u}_e + \rho_{vh}\vec{u}_h = (\rho_{ve}\mu_e + \rho_{vh}\mu_h)\vec{E} = \sigma\vec{E}$$

$\sigma = (N_e\mu_e + N_h\mu_h)\, e$ ← electron charge

density of electrons and holes

- Resistance



$$R = \frac{V}{I} = \frac{-\int_l \vec{E}\cdot d\vec{l}}{\int_s \vec{J}\cdot d\vec{s}} = \frac{-\int_l \vec{E}\cdot d\vec{l}}{\int_s \sigma\vec{E}\cdot d\vec{s}}$$

$$R = \frac{l}{\sigma A}$$

Joule's law

power dissipated in conductor:

$\Delta W = \vec{F_e} \cdot \Delta \vec{l_e} + \vec{F_h} \cdot \Delta \vec{l_h}$ energy to move charges Δl

$$\Delta P = \frac{\Delta W}{\Delta t} = \vec{F_e} \frac{\Delta \vec{l_e}}{\Delta t} + \vec{F_h} \frac{\Delta \vec{l_h}}{\Delta t} = \vec{F_e} \cdot \vec{u_e} + \vec{F_h} \cdot \vec{u_h}$$

drift velocities

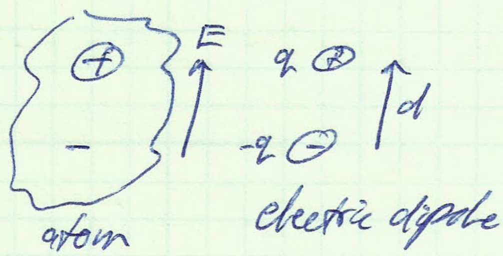
$$\Delta P = \left(\rho_{ve} \vec{E} \cdot \vec{u_e} + \rho_{vh} \vec{E} \cdot \vec{u_h} \right) \Delta V$$

total charge q in volume ΔV is charge density times ΔV

$$\Delta P = \vec{E} \cdot \vec{J} \Delta V$$

total power $P = \int_V \vec{E} \cdot \vec{J} \, dV$

Dielectrics

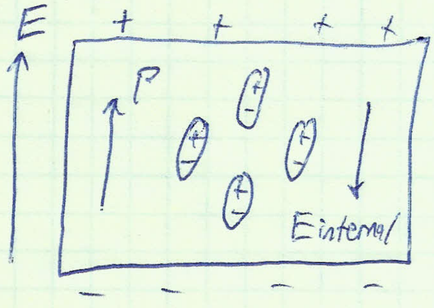


Induced electric field
called polarization field
weaker and opposite to applied field

$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$ where $\vec{P} = \epsilon_0 \chi_e \vec{E}$

electric susceptibility

$$\vec{D} = \epsilon_0 (1 + \chi_e) \vec{E} = \epsilon \vec{E}$$



Note that polarization is in
same direction as applied field
Polarized atoms create internal field
that opposes applied field